Available online at http://jprm.sms.edu.pk/
Journal of Prime Research in Mathematics, 2026, 44–50

Journal of Prime Research in Mathematics

Higher Topological Complexity of Complement of a Generic Arrangement

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Abstract

The notion of higher topological complexity of a topological space was defined by Rudyak in [12], generalizing the earlier work of M. Farber [3, 4, 5, 6]. In this paper, we compute the higher topological complexity of the complement of a generic arrangement. As an application, we compute the higher topological complexity of configuration spaces in Euclidean spaces.

Keywords: Topological complexity, otion planning algorithm, Schwarz genus, hyperplane arrangements.
2010 Mathematics Subject Classification: Primary 52C35, 55R80;

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1. Introduction

Let X be a topological space and PX the space of free paths on X and $\pi : PX \to X \times X$ the map sending each path γ to the pair of its initial and final points $\pi(\gamma) = (\gamma(0), \gamma(1))$. The topological complexity $TC(X)$ of X is defined by M. Farber in [3] as the Schwarz genus (see [13]) of π.

In [12], Y. Rudyak extended this notion to the higher topological complexity (see also [1]). For $n \geq 2$, let J_n denote the wedge of n closed unit intervals $[0,1]_i$, $i = 1, \dots, n$, where the points $0_i \in [0,1]_i$ are identified, X^{J_n} the space of all continuous maps from J_n to X with compact - open topology. Consider the map

$$\begin{array}{cccc} e_n: & X^{J_n} & \longrightarrow & X^n \quad , \\ & \gamma & \longmapsto & (\gamma(1_1), ..., \gamma(1_n)) \end{array}$$

where 1_i is the unit in $[0,1]_i$.

Definition 1 (Y. Rudyak (2010))**.** The higher topological complexity $TC_n(X)$ of the space X is the smallest integer k such that there is an open covering $U_1, \dots, U_k$ of X^n by k open sets where each U_i admits a local section $s_i : U_i \to X^{J_n}$ of e_n, i. e., $e_n \circ s_i = id_{U_i}$.

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Received : 31 December 2025*; Accepted:* 24 May 2026*; Published Online:* 31 May 2026

Note that the higher topological complexity $TC_n(X)$ of X is the Schwarz genus of the fibration e_n. The following properties of $TC_n(X)$ can be obtained from those of Schwarz genus (see [1], [12] and [13])

(i) $TC_n(X)$ is a homotopy invariant of X.

(ii) If X is a r-connected CW-complex, then

$$TC_n(X) < \frac{ndimX + 1}{r+1} + 1.$$

Particularly, $TC_n(X) \leq n \dim X + 1$ if X is path-connected.

(iii) For path-connected spaces X and Y we have $TC_n(X \times Y) \leq TC_n(X) + TC_n(Y) - 1$.

(iv) Let X be path-connected. Suppose that X^n is decomposed into $X^n = X_1 \cup ... \cup X_k$, where for any i, X_i is an ENR and there is a local section of e_n on it. Then $TC_n(X) \leq k$.

(v) If there exist $u_i \in H^*(X^n)$ for $i = 1, ..., m$ such that $d_n^* u_i = 0, \forall i$ and $u_1 \cup u_2 \cup ... \cup u_m \neq 0 \in H^*(X^n)$, then $TC_n(X) \geq m + 1$.

The higher topological complexity has been computed for spheres and product of spheres by I. Basabe, J. González, Y. Rudyak, and D. Tamaki in [1], and for wedge of spheres, Riemann surfaces and for several configuration spaces by us in [9], [10].

We are interested in computing the higher topological complexity for the complement of a hyperplane arrangement. The purpose of this paper is to determine the higher topological complexity TC_n for the complement of a generic arrangement. Generalizing the result of S. Yuzvinsky in [14] we get the following main result

Theorem *Let $\mathcal{A}$ be a generic arrangement consisting ℓ complex linear hyperplanes in $\mathbb{C}^r$, $\ell \geq r$ and $M = \mathbb{C}^r \setminus \cup_{H \in \mathcal{A}} H$ its complement. Then*

$$TC_n(M) = min\{(n-2)r + \ell + 1, nr\}.$$

Acknowledgement: The authors would like to express their sincere gratitude to the referees for their careful reading of the manuscript. Their constructive comments, suggestions and detailed corrections have helped bring the paper into its present form.

2. The higher topological complexity of complement of generic arrangement

We first recall some definitions from the theory of hyperplane arrangement. Let V be a complex vector space of dimension r. An arrangement of hyperplanes $\mathcal{A}$ in V, or an r-arrangement, is a finite collection of hyperplanes $\{H_1, \ldots, H_\ell\}$ in V. For $i = 1, \ldots \ell$, let α_{H_i} be the linear function defining H_i. The product $Q(\mathcal{A}) = \prod_{i=1}^{\ell} \alpha_{H_i}$ is called the defining polynomial of the arangement $\mathcal{A}$. An arrangement $\mathcal{A}$ is called *central* if each hyperplane contains the origin, and *essential* if $\cap_{i=1}^{\ell} H_i = \{0\}$.

For a central r-arrangement $\mathcal{A}$, we define the deconing construction as follows. Choose a hyperplane $K_0 \in \mathcal{A}$. Choose coordinates so that $K_0 = \ker(x_0)$. Suppose that the defining polynomial of $\mathcal{A}$ is $Q(\mathcal{A}) \in \mathbb{C}[x_0, x_1, \ldots, x_{l-1}]$. Then the deconing $d\mathcal{A}$ of $\mathcal{A}$ is an $(r-1)$-arrangement with $\ell - 1$ hyperplanes, having the defining polynomial $Q(d\mathcal{A})$ obtained by substituting 1 for x_0 in $Q(\mathcal{A})$.

Definition 2. An r-arrangement $\mathcal{A}$ with $r \geq 2$ is called generic if for every subset $\mathcal{B} \subseteq \mathcal{A}$ with $|\mathcal{B}| \leq r$ the set of linear functions defining hyperplanes in $\mathcal{B}$ is linearly independent.

An r-arrangement $\mathcal{A}$ is called a general position arrangement if for every subset $\{H_{i_1}, ..., H_{i_p}\} \subseteq \mathcal{A}$ with $p \leq r$, $H_{i_1} \cap ... \cap H_{i_p}$ is an affine subspace of codimension p and when $p > r$

$$H_{i_1} \cap ... \cap H_{i_p} = \emptyset.$$

If an essential arrangement $\mathcal{A}$ is generic then its deconing $d\mathcal{A}$ is a general position $(r-1)$-arrangement (see [11]).

We recall our main result.

Theorem 1. *Let* $\mathcal{A}$ *be a generic arrangement consisting of* ℓ *complex linear hyperplanes in* $\mathbb{C}^r$, $\ell \geq r$ *and* $M = \mathbb{C}^r \setminus \bigcup_{H\in\mathcal{A}} H$ *its complement. Then*

$$TC_n(M) = min\{(n-2)r + \ell + 1, nr\}.$$

Proof. **Lower bound**: It is well known that $H^*(M, \mathbb{C})$ is isomorphic to the Orlik - Solomon algebra $\mathsf{A}(\mathcal{A})$ of the arrangement $\mathcal{A}$ as graded algebras (see [11]). In order to use property (v) in the Introduction to give a lower bound for TC_n we recall the Orlik-Solomon algebra.

Let E the exterior algebra generated by generators e_H, $H \in \mathcal{A}$. Equip E with the usual differential by $\partial 1 = 0$, $\partial e_H = 1$ and for $p \geq 2$

$$\partial(e_{H_1}\dots e_{H_p}) = \sum_{k=1}^{p}(-1)^{k-1} e_{H_1}\dots \widehat{e_{H_k}} \dots e_{H_p}.$$

Let I denote the ideal of E generated by all $\partial(e_{H_1}\dots e_{H_p})$ with $codim \cap_{k=1}^{p} H_i < p$.

Then the Orlik-Solomon is defined to be the quotient $\mathsf{A}(\mathcal{A}) = E/I$.

Let denote $\varphi : E \longrightarrow \mathsf{A}(\mathcal{A})$ the natural homomorphism. Suppose that we index hyperplanes of $\mathcal{A}$ by $\{H_0, H_1, \dots, H_{\ell-1}\}$. Define $a_i = \varphi(e_{H_i})$. We can then identify the generators of $H^*(M, \mathbb{C})$ with those of $\mathsf{A}(\mathcal{A})$.

Since $\mathcal{A}$ is generic, the algebra $\mathsf{A}(\mathcal{A})$ is generated by $a_0, ..., a_{\ell-1}$ with following relations ${a_i}^2 = 0, a_i a_j = -a_j a_i$ and if $I = \{i_1, ..., i_k\} \subset \{0, 1, ..., \ell-1\}, k \leq r$ then $a_I = a_{i_1} ... a_{i_k} \neq 0$. The set of $a_I, I = \{i_1 < ... < i_k\} \subset \{0, 1, ..., \ell-1\}$ is a basis of (the vector space) $\mathsf{A}(\mathcal{A})$.

Considering a_i's as elements in $H^*(M, \mathbb{C})$, for each a_i we put

$\bar{a}_{i_t} = 1 \otimes 1 \otimes ... \otimes 1 \otimes \overset{t}{\overset{\vee}{a_i}} \otimes ... \otimes 1 - a_i \otimes 1 \otimes ... \otimes 1$, for $t = 2, ..., n$.

Let $p = min\{\ell-1, 2r-2\}$, $J = \{0, 1, ..., p\}$, $I \subset J$ with $|I| = r$.

Consider the element

$$\pi = \prod_{i\in I}\left(\prod_{t=2}^{n} \bar{a}_{i_t}\right). \prod_{i\in J\setminus I} \bar{a}_{i_n}.$$

Clearly this element π contains a non-zero term $\pm a_I \otimes ... \otimes a_I \otimes a_{J\setminus I}$ of degree $(n-2)r + p + 1$. Since this term can not be eliminated by any other summand, it follows that $\pi \neq 0$.

On the other hand, $d_n^* \bar{a}_{i_t} = 0$ for all $i = 0, 1, \dots, \ell-1, t = 2, ..., n$. So we obtain the lower bound for the higher topological complexity of the complement

$TC_n(M) \geq (n-1)r + (p+1) - r + 1 = min\{(n-2)r + \ell + 1, nr\}$.

Upper bound: We have that M is homotopy equivalent to $\mathbb{S}^1 \times M(d\mathcal{A})$ (see [11]). Since $\mathcal{A}$ is generic and $\ell \geq r$, $d\mathcal{A}$ is a general position $(r-1)$-arrangement.

Let $T^{\ell-1} = \underbrace{\mathbb{S}^1 \times \dots \times \mathbb{S}^1}_{\ell-1 \text{ times}}$ be the torus. For a subset $I \subset \{1, \dots, \ell-1\}$ we define the subtorus $T_I^{\ell-1} = \{(z_1, \dots, z_{\ell-1}) \in T^{\ell-1} \mid z_j = 1, \text{ if } j \notin I\}$ of $T^{\ell-1}$.

Then, according to Hattori Theorem (see [8]), $M(d\mathcal{A})$ is homotopy equivalent to M_0, where

$$M_0 = \bigcup_{|I|=r-1} T_I^{\ell-1}.$$

The property (iii) of TC_n gives us

$$\begin{aligned} TC_n(M) &= TC_n(\mathbb{S}^1 \times M(d\mathcal{A})) \leq TC_n(\mathbb{S}^1) + TC_n(M(d\mathcal{A})) - 1 \\ &= TC_n(\mathbb{S}^1) + TC_n(M_0) - 1. \end{aligned}$$

It is known that $TC_n(\mathbb{S}^1) = n$ (see[12]).

Since M_0 is the $(r-1)$-skeleton of the canonical CW-complex of $T^{\ell-1}$, then $TC_n(M_0) \le n(r-1)+1$.

It implies $TC_n(M) \le n + (n(r-1)+1) - 1 = nr$.

If $\ell + 1 \ge 2r$, we have $TC_n(M) \le nr = min\{(n-2)r + \ell + 1, nr\}$ and we are done.
So, we need only to consider the case $\ell + 1 < 2r$.

Claim: $TC_n(M_0) \le (n-2)(r-1) + \ell$.
To demonstrate this, we decompose $(M_0)^n$ into union of $(n-2)(r-1)+\ell$ ENR subsets and construct a local section on each of these subsets.
First, for $J \subset \{1,\dots,\ell-1\}$, $I \subset \{2,\dots,n-1\}$ consider following subsets of $(T^{\ell-1})^n$

$F'_J = \{(u_1, u_2, \dots, u_n) \mid u_{1j} = u_{2j} = \dots = u_{nj}$ if and only if $j \in J\}$

$U'_{IJ} = \{(u_1, u_2, \dots, u_n) \mid u_{1j} = u_{ij}$ if and only if $i \in I, j \in J\}$.

Put:

$F_0 = F'_0 \cap (M_0)^n$, where $F'_0 = \{(u_1, u_2, \dots, u_n) \mid u_{1j} \ne u_{ij}$ for all $i = 2,\dots,n, j = 1,\dots,\ell-1\}$,

$F_j = \bigcup_{|J|=j} F_J,\ \ j = 1,\dots,\ell-1$, where $F_J = F'_J \cap (M_0)^n$,

$U_{st} = \bigcup_{|I|=s,|J|=t} U_{IJ},\ \ s = 1,\dots,n-2, t = 1,\dots,r-1$, where $U_{IJ} = U'_{IJ} \cap (M_0)^n$.

By defintion, $(M_0)^n$ is decomposed into disjoint union of $(n-2)(r-1)+\ell$ subsets F_j, $j = 0,1,\dots,\ell-1$ and U_{st}, $s = 1,\dots,n-2, t = 1,\dots,r-1$). We will construct a (continuous) section of e_n on each of these subsets. Observe that each of F_j is disjoint union of F_J and each U_{st} is disjoint union of U_{IJ}. So, we need only to construct a section on each of F_J, U_{IJ}.

First for $u = (u_1,\dots,u_{\ell-1}), u' = (u'_1,\dots,u'_{\ell-1}) \in M_0$ we define a path $l_{(u,u')}(t) = (l_{(u,u')}(t)_1,\dots,l_{(u,u')}(t)_{\ell-1})$ in M_0 going from u to u' by defining its $j-th$ coordinate $l_{(u,u')}(t)_j$.

- If $u_j = u'_j$ we define $l_{(u,u')}(t)_j = u_j = u'_j$ for all $t \in [0,1]$.

- Suppose that $u_j \ne u'_j$. For two distinct points $z = e^{i\varphi}, z' = e^{i\varphi'}$ with $0 \le \varphi, \varphi' < 2\pi$ on the circle $\mathbb{S}^1$, let $\eta_{z,z'}$ denote the canonical path on the circle going from the point z to the point z' with constant speed. Precisely, η is given by $\eta_{z,z'}(t) = e^{i((1-t)\varphi + t\varphi')}, t \in [0,1]$. Since $u_j \ne u'_j$ are two distinct points on the same circle $\mathbb{S}^1$ we have the path η_{u_j,u'_j} on the circle connecting the point u_j to the point u'_j.

Considering $\mathbb{S}^1$ as the set of complex numbers of modulus 1, we define the map $\tau : \mathbb{S}^1 \to [0, 1/2]$ by

$$\tau(z) = \begin{cases} 1/2(1 - (|z-1|)/\sqrt{2}) & \text{if } |z-1| \le \sqrt{2} \\ 0 & \text{otherwise.} \end{cases}$$

Using this map τ as the reparameterization function we define the *j-th* coordinate $l_{(u,u')}(t)_j$ for $u \ne u'$ by

$$l_{(u,u')}(t)_j = \begin{cases} u_j & \text{if } 0 \le t < \tau(u_j); \\ \eta_{u_j,u'_j}\left(\dfrac{t - \tau(u_j)}{1 - \tau(u_j) - \tau(u'_j)}\right) & \text{if } \tau(u_j) \le t < 1 - \tau(u'_j); \\ u'_j & \text{if } 1 - \tau(u'_j) \le t \le 1. \end{cases}$$

Obviously, $l_{(u,u')}(t)$ is well defined for all $u, u' \in T^{l-1}$ and is a continuous path going from $l_{(u,u')}(0) = u$ to $l_{(u,u')}(1) = u'$.

Next, we need to verify that the image of $l_{(u,u')}$ lies completely inside M_0 if $u, u' \in M_0$. That is for every $t \in [0,1]$, $l_{(u,u')}(t)$ contains at least $\ell - r$ coordinates 1.

Assume $u \in T_I^{\ell-1}$ and $u' \in T_{I'}^{\ell-1}$, where $I, I' \subset \{1,...,\ell-1\}$ satisfying $|I| = |I'| = r-1$. Denote $\bar{I} = \{1,...,\ell-1\} \backslash I$, $\bar{I}' = \{1,...,\ell-1\} \backslash I'$ and put $I_0 = \bar{I} \cap \bar{I}'$. So, we have $|\bar{I}| = |\bar{I}'| = \ell - r$

For any $t \in [0,1]$ we consider indices $j \in I_0$. Since $j \notin I$ and $j \notin I'$ we have $u_j = u'_j = 1$ and by definition $l_{(u,u')}(t)_j = u_j = u'_j = 1$ for all $t \in [0,1]$.

For $t \in [0,1/2]$ we consider indices $j \in \bar{I} \backslash I_0$. Since $j \notin I$ we have $u_j = 1$. Therefore, $\tau(u_j) = \frac{1}{2}$ and by definition $l_{(u,u')}(t)_j = u_j = 1$ for all $t \in [0, \frac{1}{2}]$

For $t \in [1/2,1]$ we consider indices $j \in \bar{I}' \backslash I_0$. Since $j \notin I'$ we have $u'_j = 1$. Therefore, $\tau(u'_j) = \frac{1}{2}$ and by definition $l_{(u,u')}(t)_j = u'_j = 1$ for all $t \in [\frac{1}{2}, 1]$

That is for every $t \in [0,1]$, $l_{(u,u')}(t)$ has at least $\ell - r$ coordinates equal 1, or equivalently, $l_{(u,u')}$ lies completely inside M_0.

Now we define the map : $s : M_0^n \to M_0^{J_n}$ by

$$s(u_1, u_2, ..., u_n) = (l_{(u_1,u_1)}, l_{(u_1,u_2)}, ..., l_{(u_1,u_n)}).$$

By definition, the function τ is continuous, the path $\eta_{(z,z')}$ depends continuously on distinct pairs (z, z') on $\mathbb{S}^1 \times \mathbb{S}^1$. Therefore, the restrictions of s on F_J, F_0, U_{IJ} depend continuously on $(u_1, u_2, ..., u_n)$ and define local sections of e_n on F_j, U_{st}.

Moreover, the sets F_j, U_{st} for $j = 0, ..., \ell - 1, s = 1, ..., n-2, t = 1, ..., r-1$ are all $ENRs$. So, we have $TC_n(M_0) \leq (n-2)(r-1) + \ell$. That is

$$TC_n(M) \leq n + (n-2)(r-1) + \ell - 1 = (n-2)r + \ell + 1.$$

Since $\ell + 1 < 2r$, we have $TC_n(M) \leq min\{(n-2)r + \ell + 1, nr\}$.

So, $TC_n(M) = min\{(n-2)r + \ell + 1, nr\}$. □

3. Applications to configuration spaces

In this section, using the result of the previous section we will compute the higher topological complexity TC_n of configuration space on Euclidean space $\mathbb{R}^m$ with even $m \geq 2$.

First, we consider the case when $m = 2$.

Theorem 2. *Let $F(\mathbb{R}^2, k)$ be the configuration space of k distinct points on $\mathbb{R}^2$, with $k \geq 2$. Then*

$$TC_n(F(\mathbb{R}^2; k)) = n(k-1).$$

Proof. Considering $\mathbb{R}^2$ as the complex line $\mathbb{C}$ we can identify $F(\mathbb{R}^2; k)$ with the complement $M(\mathcal{A})$ of braid arrangement $\mathcal{A} = \{H_{ij} | H_{ij} : x_i - x_j = 0, 1 \leq i < j \leq k\}$ in $\mathbb{C}^k$. The hyperplane $H_0 : x_1 + ... + x_k = 0$ is a complex vector space of dimension $k-1$ and $\mathcal{A}_0 := \{H_{ij} \cap H_0 \mid H_{ij} \in \mathcal{A}\}$ is a generic arrangement in H_0 consisting $\dfrac{k(k-1)}{2}$ hyperplanes. Moreover, $M(\mathcal{A}_0)$ is a retract of $M(\mathcal{A})$. Then the property of TC_n gives $TC_n(M(\mathcal{A})) = TC_n(M(\mathcal{A}_0))$. Hence Theorem 1 implies

$TC_n(M(\mathcal{A}_0)) = min\{(n-2)(k-1) + \frac{k(k-1)}{2} + 1; n(k-1)\} = n(k-1)$ for all $k \geq 2$.

So, $TC_n(F(\mathbb{R}^2; k)) = n(k-1)$. □

Lemma 1. *Let X be a $(s-1)$-connected k-dimensional finite cell complex where $s \geq 2$. Assume additionally that $nk = rs$ where $r > 0$ is an integer. Then*

i) $TC_n(X) \leq r + 1$,

ii) $TC_n(X) = r + 1$ if only if there exist $u_i \in H^s(X^n)$ for $i = 1, ..., r$ such that $d_n^* u_i = 0, \forall i$ and $u_1 \cup u_2 \cup ... \cup u_r \neq 0 \in H^*(X^n)$.

Proof. Statement *i*) follows directly from the inequality

$$TC_n(X) < \frac{nk+1}{s} + 1$$

for any $(s-1)$-connected CW-complex X of dimension k.

Statement *ii*) follows from Theorem 6 in [13] by considering TC_n as Schwarz genus of the fibration e_n. □

A computation for the higher topological complexity of $TC_n(F(\mathbb{R}^m,k))$ for any $m \geq 3$ has been presented by us in [9] and independently in [2]. In this section, we present an alternative computation for the case of even m, using our results on the TC_n of the complement for a generic arrangement in Theorem 1 and Theorem 2.

Theorem 3. *Let $F(\mathbb{R}^m,k)$ be the configuration space of k distinct points on $\mathbb{R}^m$, $k \geq 2$. Then*

$$TC_n(F(\mathbb{R}^m;k)) = \begin{cases} n(k-1) & \textit{if } m \textit{ even} \\ n(k-1)+1 & \textit{if } m \textit{ odd}. \end{cases}$$

Proof. The result for the case m odd has been given in [9] and for $m=2$ in the Theorem 2. So we need only to consider m even, $m \geq 4$.

Since $m \geq 4$ the space $F(\mathbb{R}^m,k)$ has the homotopy type of a polyhedron of dimension $\leq (k-1)(m-1)$, hence $(m-2)$- connected (see [7]). Using the property (ii), we have

$$TC_n(F(\mathbb{R}^m,k)) < \frac{n(k-1)(m-1)+1}{m-1} + 1 = n(k-1) + \frac{1}{m-1} + 1.$$

It implies $TC_n(F(\mathbb{R}^m,k)) \leq n(k-1)+1$.
Therefore, by means of the Lemma 1, $TC_n(F(\mathbb{R}^m,k)) = n(k-1)+1$ if and only if there exist cohomology classes

$$u_1,...,u_{n(k-1)} \in H^{m-1}((F(\mathbb{R}^m,k))^n)$$

such that $d_n^* u_i = 0$ and $u_1 \cup ... \cup u_{n(k-1)} \neq 0$.
It is known that for any even $m \geq 2$, there is an algebra isomorphism (see [6])

$$\phi : H^*(F(\mathbb{R}^2,k)) \to H^{(*(m-1))}(F(\mathbb{R}^m,k))$$

sending classes of degree i to classes of degree $i(m-1)$ where $i = 0,1,...,k-1$.
On the other hand, combining Lemma 1 and Theorem 2 we see that there is no $n(k-1)$ cohomology classes in $H^*(F(\mathbb{R}^2,k)^n)$ satisfying above conditions. So, there does not exist $n(k-1)$ cohomology classes $u_1,...,u_{n(k-1)}$ in $H^{(m-1)}((F(\mathbb{R}^m,k))^n)$ such that $d_n^* u_i = 0$, $i = 1,..,n(k-1)$ and $u_1 \cup ... \cup u_{n(k-1)} \neq 0$. Hence $TC_n(F(\mathbb{R}^m,k)) < n(k-1)+1$.

It is well known, see [7], that $H^*(F(\mathbb{R}^m,k))$ is generated by classes $e_{ij} \in H^{m-1}(F(\mathbb{R}^m,k)), 1 \leq i \neq j \leq k$ satisfying $e_{ij}^2 = 0$, $e_{ij}e_{jt} + e_{jt}e_{ti} + e_{ti}e_{ij} = 0$ for all $1 \leq i,j,t \leq k$.

For every $t = 2,...,n$, we put

$$\bar{e}_{ij_t} = 1 \otimes 1 \otimes ... \otimes 1 \otimes \overset{\overset{t}{\vee}}{e_{ij}} \otimes ... \otimes 1 - e_{ij} \otimes 1 \otimes ... \otimes 1$$

$$\bar{e}_{ij} = \prod_{t=2}^{n} \bar{e}_{ij_t} = \sum_{l=1}^{n} (-1)^{l-1} e_{ij} \otimes e_{ij} \otimes ... \otimes \overset{\overset{l}{\vee}}{1} \otimes e_{ij} \otimes ... \otimes e_{ij}.$$

Then, by arguments similar to those in the proof for the lower bound of Theorem 1, the element

$$\prod_{j=2}^{k} \bar{e}_{1j} \prod_{p=3}^{k} \bar{e}_{2p_n} = \prod_{j=2}^{k} (\prod_{t=2}^{n} \bar{e}_{1j_t}) \prod_{p=3}^{k} \bar{e}_{2p_n}$$

is nonzero. Moreover $d_n^* \bar{e}_{ij_t} = 0$ for all $1 \leq i \neq j \leq k, t = 2,...,n$. Using the cohomological lower bound, we get

$$TC_n(F(\mathbb{R}^m,k) \geq (n-1)(k-1) + (k-2) + 1 = n(k-1).$$

Thus, $TC_n(F(\mathbb{R}^m,k)) = n(k-1)$. □

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