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Spin Squeezing, Entanglement, and Intelligent Coherent States in a Spin-1 System Interacting with an External Magnetic Field

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Abstract

In this paper, we study a spin-1 system's coherent states in the context of the f -deformed $\mathfrak{su}_f(2)$ algebra. This algebraic framework is used to generate intelligent coherent spin states, and the accompanying uncertainty relations are studied to analyze their quantum features. Nonlinear effects related to absorption saturation and dispersion phenomena are shown by calculating the coherent states of the entire spin system up to third-order approximation. The resulting asymptotic formulas show that the excitation frequency and spin state transition processes have a significant influence on the nonlinear behavior. Additionally, we demonstrate that the suggested states have squeezing characteristics typical of intelligent states and meet minimum uncertainty relations. Additionally included are the impact of relaxation effects, material-dependent factors, and higher-order corrections on the nonlinear dynamics. These findings could lead to new advances in quantum information processing, nonlinear quantum systems, and quantum optics.

Keywords: $\mathfrak{su}_f(2)$ algebra, spin systems, coherent states, intelligent states, Schrödinger equation, nonlinear coherent states, and quantum information.

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1. Introduction

Coherent states are quantum states whose dynamical behavior most closely mimics that of classical systems. They were first introduced within the framework of the quantum harmonic oscillator. Due to their intimate relationship to the semiclassical description of quantum systems, coherent states are among the most significant ideas in contemporary quantum physics. Numerous applications in quantum optics, atomic physics, condensed matter physics, quantum information, and quantum field theory have resulted from their exceptional mathematical and physical characteristics. A strong theoretical foundation for researching

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quantum fluctuations, uncertainty relations, and nonclassical phenomena has been made possible by the generalization of coherent states to Lie groups and finite-dimensional quantum systems. The description of angular momentum systems relies heavily on spin coherent states connected to the $\mathfrak{su}_f(2)$ algebra among various generalized formulations. These states are especially helpful when studying spin fluctuations and quantum correlations in finite-dimensional Hilbert spaces.

The Heisenberg uncertainty relation's equality condition, which asserts that the uncertainties are ideally distributed between two orthogonal spin components perpendicular to the mean spin direction, is satisfied by such states. Intelligent states are thought to be the best options for interferometric applications and high-precision quantum measurements because of their lowest uncertainty characteristics.

The study of entanglement and quantum correlations in many-body quantum systems has received a lot of attention in recent decades [1, 2]. Because of its uses in quantum metrology, quantum communication, and quantum computing, spin-squeezed states have become especially significant quantum states in this context. The reduction of quantum fluctuations in one spin component below the conventional quantum limit at the cost of enhanced fluctuations in another conjugate component is known as spin squeezing. This phenomenon is essential to the development of next-generation quantum technologies because it enables measurements with a precision that exceeds the usual quantum limit. Numerous physical systems, such as confined ions, atomic ensembles, and Bose–Einstein condensates, have reported experimental realizations of spin-squeezed states [3, 4, 5, 6, 7]. Specifically, a nonlinear Ramsey interferometer produced a significant experimental proof of interferometric phase precision beyond the usual quantum limit [4]. Coherent spin-squeezed states were found in a Bose–Einstein condensate of rubidium atoms (^{87}Rb). The “one-axis twisting” process presented by Kitagawa and Ueda [8] served as the basis for the nonlinear atomic beam splitter used in the experiment. This mechanism has grown to be a key paradigm in the investigation of spin squeezing and collective quantum fluctuations.

Aragnone and associates first proposed the idea of intelligent states in the context of the $\mathfrak{su}_f(2)$ algebra [15]. Since then, a number of theoretical strategies for creating cohesive intelligent states have been put forth. Rashid presented a design based on nonunitary transformations [2], whereas polynomial representations for $\mathfrak{su}_f(2)$ intelligent states were created in [3]. Coherent intelligent states for the $\mathfrak{su}_f(2)$ algebra can be created using algebraic techniques and recursive procedures [4]. These states' interferometric characteristics and uncertainty minimization have also been thoroughly studied [5]. Furthermore, measures of angular uncertainty and circular compressed states associated with intelligent behavior were examined in [14].

Deformed coherent states have been introduced in the context of quantum groups and deformed algebras as a natural extension of coherent states associated with Lie algebras. These generalized states offer a useful mathematical tool for characterizing nonclassical phenomena and nonlinear quantum systems. Specifically, the usual commutation relations are modified nonlinearly by the f -deformed algebra $\mathfrak{su}_f(2)$, resulting in new physical effects and generalized quantum fluctuations. Because they enable the study of nonlinear quantum dynamics, quantum statistical characteristics, and generalized squeezing events outside of the conventional algebraic framework, deformed coherent states have garnered growing attention.

In this work, we study the uncertainty relations related to the deformed algebra $\mathfrak{su}_f(2)$ for f -deformed coherent spin states, inspired by the groundbreaking work of Kitagawa and Ueda [8]. Our primary goal is to investigate the angular momentum component fluctuations perpendicular to the mean spin direction and show how these states behave intelligently by minimizing the corresponding uncertainty connections. The study of spin squeezing, nonlinear quantum effects, and how the deformation function affects the system's statistical and physical characteristics will receive special emphasis.

We also study how deformation affects uncertainty distributions and quantum correlations. According to the observed results, f -deformed coherent spin states have nonclassical characteristics that could be applied in quantum information processing, quantum metrology, and quantum interferometry. Additionally, the nonlinear deformation offers more control over squeezing features and quantum fluctuations, which could open up new avenues for research on finite-dimensional quantum systems.

The production of the appropriate deformed coherent spin states and the mathematical formalism related to the $\mathfrak{su}_f(2)$ algebra are presented in the first section. Next, we apply the theory to the total spin ($S =$

1) analytically solvable case. The properties of spin fluctuations, squeezing, and intelligent behavior are discussed and numerical and analytical conclusions for the general situation are presented in the subsequent sections. The final section concludes by summarizing the key findings and outlining potential directions for further research. Lastly, we observe that intelligent states are occasionally referred to as “minimum uncertainty states” in the literature [11, 12, 13, 14, 15]. We steer clear of this nomenclature, though, because it is always possible to select a normalized eigenstate of a given observable in finite-dimensional representations so that the accompanying uncertainty vanishes identically. As a result, the phrase “intelligent states” seems more fitting in this situation [16, 17, 18, 19, 20].

2. Mathematical Formalities

This part is devoted to preparing the mathematical formalities necessary to define the intelligent coherent state. We study the behavior of uncertainty relations for a coherent intelligent state, and we show some analytical results for the case of spin $S = 1$.

2.1. Intelligent coherent states for spin

The ability of intelligent states to attain the lowest level of uncertainty permitted by quantum physics makes them a noteworthy class of quantum states. Because they saturate the Heisenberg uncertainty relation, these states are crucial to the study of quantum fluctuations and coherence. Quantum states for which the uncertainty product of two non-commuting spin observables achieves its theoretical bottom bound are known as intelligent coherent states in spin systems. These states show the best possible equilibrium between the relevant spin components’ quantum fluctuations. The simultaneous measurement of two spin components is fundamentally constrained by the Heisenberg uncertainty principle when taking into account the angular momentum operators S_1 , S_2 , and S_3 , which fulfill the $\mathfrak{su}_f(2)$ commutation relations. The uncertainty relation for the set of observables S_1 and S_2 is written as:

$$\Delta S_1^2 \Delta S_2^2 \geq \frac{1}{4} |\langle S_3 \rangle|^2 \quad (2.1)$$

where $\langle S_3 \rangle$ is the expectation value of the third spin operator and $\Delta S_i^2 = \langle S_i^2 \rangle - \langle S_i \rangle^2$ ($i = 1, 2$) is the variance of the associated spin component. The states for which the equality sign is true are intelligent coherent spin states, with $\Delta S_1^2 \Delta S_2^2 = \frac{1}{4} |\langle S_3 \rangle|^2$, demonstrating that the distribution of quantum uncertainty is as efficient as allowed by the laws of quantum physics. As a result, these states offer a suitable framework for studying non-classical effects, quantum correlations, and squeezing phenomena in finite-dimensional spin systems.

The fulfillment of condition (1) occurs if and only if

$$\langle \{S_1 - \langle S_1 \rangle, S_2 - \langle S_2 \rangle\} \rangle = 0 \quad (2.2)$$

$$(S_1 - \langle S_1 \rangle) |\Psi\rangle = ix(S_2 - \langle S_2 \rangle) |\Psi\rangle \quad (2.3)$$

where x is a real number. This is equivalent to finding the eigenvectors of the non-Hermitian operator [1] $S_x = S_1 - ixS_2$, namely:

$$S_x |n\rangle = n |n\rangle \quad (2.4)$$

where n denotes the corresponding eigenvalue.

The intelligent coherent spin states can be described as follows using the approach suggested in Ref. [1]:

$$|\Psi\rangle = |S, m\rangle_x = N_{Smx} e^{\alpha S_3} e^{-i\frac{\pi}{2} S_2} |S, m\rangle \quad (2.5)$$

with N_{Smx} a normalization constant and α :

$$\alpha = \frac{1}{2} \ln \left(\frac{1-x}{1+x} \right) \quad (-1 < x < 1).$$

The spin coherent state's deformation is described by the parameter α , which is closely correlated with the squeezing parameter x .

The spin operator variances adopt very straightforward forms for the intelligent spin states mentioned above. One acquires

$$\Delta^2 S_1 = \frac{1}{2} |x \langle S_3 \rangle| \quad (2.6)$$

and

$$\Delta^2 S_2 = \frac{1}{2} \left| \frac{x \langle S_3 \rangle}{2} \right| \quad (2.7)$$

These formulas demonstrate the inverse relationship between the two transverse spin components' fluctuations. The uncertainty product stays at its lowest value when one component's uncertainty falls and the other's uncertainty rises.

It is convenient to introduce the dimensionless parameters in order to measure these relative fluctuations:

$$\xi_1^2 = \frac{2 \Delta^2 S_1}{|\langle S_3 \rangle|} \quad (2.8)$$

and

$$\xi_2^2 = \frac{2 \Delta^2 S_2}{|\langle S_3 \rangle|} \quad (2.9)$$

When Eqs. (6) and (7) are substituted into the aforementioned definitions, the resulting parameters satisfy

$$\xi_1^2 = |x| \quad (2.10)$$

and

$$\xi_2^2 = \frac{1}{|x|} \quad (2.11)$$

Consequently, $\xi_2^2 \xi_1^2 = 1$. It shows the precise equilibrium between the two orthogonal spin components' fluctuations, and they clearly satisfy the equation $\xi_2^2 \xi_1^2 = 1$. Indeed, one could consider the parameters ξ_2^2 and ξ_1^2 to be comparable to the spin-squeezing ones found in [8].

Nevertheless, the values ξ_2^2 and ξ_1^2 are typically not invariant by $\text{su}_f(2)$ transformations, in contrast to the squeezing parameters described in the literature study [8]. The standard deviation of the transverse spin components and the expected value of the overall spin vector are typically used to describe the classic squeezing parameters as ratios [11, 23]. The parameters presented here, on the other hand, merely describe the relative fluctuations of the components S_1 and S_2 in relation to the mean value of S_3 . Consequently, rather than being true $\text{su}_f(2)$ invariant squeezing parameters, they should be thought of as measurements of anisotropic quantum fluctuations [25, 26, 27].

It is possible to estimate the third spin component's expected value explicitly. Following simple computations, one gets

$$\begin{aligned} \langle S_3 \rangle = N_{S_{mx}}^{-2} \frac{1}{2} f(\alpha)^{2m} & \left(f(\alpha) \frac{(S-m)!}{(S+m)!} \sum_k \frac{F(\alpha)^{2k+1} (S+m+k+1)!}{k! (k-1)! (S-m-k-1)!} \right. \\ & \left. + f(\alpha)^{-1} \frac{(S-m-1)!}{(S+m)!} \sum_k \frac{F(\alpha)^{2k-1} (S+m+k-1)!}{k! (k-1)! (S-m-k+1)!} \right) \end{aligned} \quad (2.12)$$

The normalizing condition ${}_x \langle S, m | S, m \rangle_x = 1$ is used to determine the normalization constant $N_{S_{mx}}$. This results in

$$N_{S_{mx}}^{-2} = f(\alpha)^{2m} \left(\frac{(S-m)!}{(S+m)!} \sum_k \frac{F(\alpha)^{2k} (S+m+k)!}{k! (S-m-k)!} \right) \quad (2.13)$$

The preceding expressions' auxiliary functions are defined by

$$f(\alpha) = \tanh(\alpha) \quad (2.14)$$

and

$$F(\alpha) = \cosh(\alpha) \quad (2.15)$$

The expectation value of the spin projection along the third axis is fully determined by equations (12)–(15), which also supply the basic quantities needed to examine squeezing, uncertainty relations, and nonclassical characteristics of intelligent coherent spin states.

2.2. f -deformed coherent states

The uncertainty relations related to the f -deformed coherent spin states are examined in this subsection. These states are built in the context of the quantum algebra $\mathfrak{su}_f(2)$, which is a deformation of the traditional $\mathfrak{su}_f(2)$ Lie algebra by adding a deformation parameter f . The generators distorted commutation relations are obeyed by S_+ , S_- , and S_z [28, 29, 30, 31, 32, 33]:

$$[S_z, S_{\pm}] = \pm S_{\pm}, \quad (2.16)$$

$$[S_+, S_-] = [2S_z]_f \quad (2.17)$$

where the f -deformed number is shown by $[x]_f$. The statistical characteristics of the related quantum states are impacted by the deformation, which alters the algebraic structure of the spin operators. Specifically, the squeezing properties, uncertainty relations, and quantum fluctuations are different from those of regular spin coherent states.

The f -number is defined by

$$[x]_f = \frac{\sin(zx)}{\sin(z)} \quad (2.18)$$

where the relationship between the deformation parameter and z is given by

$$f = e^{iz} \quad (2.19)$$

The spin quantum number S characterizes the irreducible representation of the algebra $\mathfrak{su}_f(2)$. Similar to the undeformed scenario, the orthonormal basis is provided by $\{|S, m\rangle, m = -S, -S+1, \dots, S\}$. The action of the generators on this basis reads

$$S_z |S, m\rangle = m |S, m\rangle \quad (2.20)$$

$$S_+ |S, m\rangle = \sqrt{[S-m]_f [S+m+1]_f} |S, m+1\rangle \quad (2.21)$$

$$S_- |S, m\rangle = \sqrt{[S+m]_f [S-m+1]_f} |S, m-1\rangle \quad (2.22)$$

Applying the f -exponential operation to the lowest-weight state $|S, -S\rangle$ yields the f -deformed coherent spin states:

$$|N\rangle = N e_f^{\mu S_+} |S, -S\rangle \quad (2.23)$$

where N is a normalization factor and $\mu \in \mathbb{C}$ is the coherence parameter.

The definition of the f -exponential function is

$$e_f^x = \sum_k \frac{x^k}{[k]_f!} \quad (2.24)$$

with

$$[k]_f! = [k]_f [k-1]_f \cdots [1]_f, \quad [0]_f! = 1 \quad (2.25)$$

The state's expansion (23) provides

$$|N\rangle = N \sum_{k=0}^{2S} \frac{\mu^k}{[k]_f!} (S_+)^k |S, -S\rangle \quad (2.26)$$

Using the raising operator's repeated action,

$$(S_+)^k |S, -S\rangle = \sqrt{\frac{[2S]_f!}{[k]_f! [2S-k]_f!}} |S, -S+k\rangle \quad (2.27)$$

one is able to write

$$|N\rangle = \mathcal{N} \sum_{k=0}^{2S} \mu^k \sqrt{[2Sk]_f} |S, -S+k\rangle \quad (2.28)$$

where

$$[nk]_f = \frac{[n]_f!}{[k]_f! [n-k]_f!} \quad (2.29)$$

is the f -deformed binomial coefficient.

The condition of normalizing $\langle N|N\rangle = 1$ results in

$$\mathcal{N}^{-2} = \sum_{k=0}^{2S} |\mu|^{2k} [2Sk]_f \quad (2.30)$$

Consequently,

$$N = \frac{1}{\sqrt{(1 + |\mu|^2)_f^{2S}}} \quad (2.31)$$

where

$$(1 + |\mu|^2)_f^{2S} = \sum_{k=0}^{2S} |\mu|^{2k} [2Sk]_f \quad (2.32)$$

The transverse spin components are determined by

$$S_x = \frac{S_+ + S_-}{2} \quad (2.33)$$

$$S_y = \frac{S_+ - S_-}{2i} \quad (2.34)$$

The commutation relation (17) yields

$$[S_x, S_y] = \frac{i}{2} [2S_z]_f \quad (2.35)$$

The generalized Heisenberg uncertainty relation so becomes

$$\Delta^2 S_x \Delta^2 S_y \geq \frac{1}{4} |\langle [S_x, S_y] \rangle|^2 = \frac{1}{16} \left| \langle [2S_z]_f \rangle \right|^2 \quad (2.36)$$

The groundbreaking investigations of coherent spin states by Roy J. Glauber and its application to angular momentum systems by Francis T. Arecchi and associates [20, 21, 22, 23, 24, 25] are the source of the concept of adding a coordinate system aligned with the mean spin direction. After Masahiro Kitagawa and Masahiro Ueda revealed spin squeezing as a characteristic of nonclassical correlations, this method gained significant attention. Since then, the standard framework for examining quantum fluctuations, squeezing, and entanglement in spin systems has been the rotational frame described by the mean spin vector [26, 27, 28, 29, 30]. It is convenient to construct a rotated coordinate system whose new z -axis is aligned with the

mean spin direction in order to study the squeezing qualities of a spin state. The quantum fluctuations are examined in the perpendicular plane in this reference frame, whereas the average spin points along a single axis. Since the reduction of quantum noise can be directly examined in relation to the mean spin direction, this method offers a natural and effective technique to define spin squeezing. The normalized mean spin vector defines the appropriate unit vector [32, 33, 34, 35, 36]:

$$\rho_z = \frac{\langle \mathbf{S} \rangle}{|\langle \mathbf{S} \rangle|} \quad (2.37)$$

with $\langle \mathbf{S} \rangle = (\langle S_x \rangle, \langle S_y \rangle, \langle S_z \rangle)$.

The uncertainty relation in this rotated frame becomes

$$\Delta^2 S_{\rho_x} \Delta^2 S_{\rho_y} \geq \frac{1}{4} |\langle [S_{\rho_x}, S_{\rho_y}] \rangle|^2 \quad (2.38)$$

The squeezing parameters are defined by Kitagawa and Ueda:

$$\xi_x'^2 = \frac{2(\Delta S_{\rho_x})^2}{|\langle [S_{\rho_x}, S_{\rho_y}] \rangle|} \quad (2.39)$$

$$\xi_y'^2 = \frac{2(\Delta S_{\rho_y})^2}{|\langle [S_{\rho_x}, S_{\rho_y}] \rangle|} \quad (2.40)$$

Spin squeezing is said to occur anytime

$$\xi_x'^2 < 1 \quad \text{or} \quad \xi_y'^2 < 1 \quad (2.41)$$

Important quantitative markers of the quantum correlations and nonclassical characteristics displayed by the f -deformed coherent spin states are provided by the expectation values, variances, and uncertainty relations derived above. The statistical moments of the relevant observables are impacted by the modification of the algebraic structure of the spin operators by the deformation function f . Therefore, depending on its value, the deformation parameter can either promote or suppress spin squeezing and create major changes in the distribution of quantum fluctuations. These changes disclose richer nonclassical behavior and differentiate the f -deformed coherent states from the traditional $\mathfrak{su}_f(2)$ coherent states. As a result, the deformation parameter is essential in regulating the system's squeezing properties, uncertainty relations, and quantum correlations, offering more flexibility for the study of nonclassical processes in finite-dimensional spin systems [37, 38, 39].

3. The $\mathfrak{su}_f(2)$ Formalism Is Used to Create the Intelligent Coherent States of a Spin System Interacting with a Magnetic Field

3.1. System examined

In this portion, we examine a quantum system made up of a particle interacting with an external magnetic field and having spin $S = 1$. The spin-1 particle has three magnetic sublevels that correspond to the spin operator S_z 's eigenstates $|1\rangle$, $|0\rangle$, and $|-1\rangle$. These states constitute a comprehensive basis of the three-dimensional Hilbert space and are distinguished by their individual energies in the absence of interaction [40, 41].

When an external magnetic field is introduced, the particle's magnetic moment interacts with the field, changing the energy spectrum and causing transitions between the various spin projections. The Hamiltonian controls the system's dynamics:

$$H = H_0 + H_{\text{int}} \quad (3.1)$$

with

$$H = \hbar\omega_0 I_z + 2\hbar\omega_1 I_x \cos \omega t \quad (3.2)$$

$$H_0 = \hbar\omega_0 I_z \quad (3.3)$$

$$H_{\text{int}} = 2\hbar\omega_1 I_x \cos \omega t \quad (3.4)$$

where H_{int} represents the interaction between the spin and the magnetic field and H_0 stands for the free-spin Hamiltonian. The interaction Hamiltonian for a time-dependent magnetic field can be expressed in terms of the spin operators S_x , S_y , and S_z , which cause coherent superpositions of quantum states and transitions between the magnetic sublevels.

We construct the Hamiltonian perturbatively up to third order in the interaction parameter in order to examine the implications of the spin-field coupling beyond the linear regime. Single-spin transitions that are directly caused by the magnetic field are described by the first-order approximation. Indirect transition pathways and interference effects between several excitation channels are taken into consideration by the second-order contribution [42, 43, 44]. Larger-order nonlinear processes are included in the third-order correction; these processes become important at larger coupling strengths or longer interaction timeframes.

These higher-order terms cause a redistribution of populations among the spin levels by altering the probability amplitudes linked to the states $|1\rangle$, $|0\rangle$, and $|-1\rangle$. As a result, the resulting coherent states take on a nonlinear nature that is very different from the normal spin coherent states. As a result, the magnetic field's influence is seen not only in the spin's precession but also in the creation of nonclassical processes as quantum correlations, spin squeezing, and changed uncertainty relations.

A strong framework for examining the development of intelligent coherent states and comprehending how nonlinear spin dynamics arise from the interaction between the spin-1 system and the external magnetic field is provided by the perturbative formulation of the Hamiltonian up to third order [44, 45, 46].

3.2. First-Order Approximation for a Spin-1 System under the $\text{su}_f(2)$ Formalism

A spin-1 quantum system interacting with an external magnetic field is examined in this section. The generators S_x , S_y , and S_z , which satisfy the basic commutation relations, define the system inside the $\text{su}_f(2)$ framework:

$$[S_i, S_j] = i\hbar \varepsilon_{ijk} S_k \quad (3.5)$$

The basis of the irreducible representation of the formalism method $\text{su}_f(2)$ with total spin $S = 1$ is the Hilbert space, which is spanned by the eigenstates of S_z , namely, $\{|1\rangle, |0\rangle, |-1\rangle\}$.

The interaction Hamiltonian causes transitions between neighboring spin projections by coupling these magnetic sublevels through the ladder operators $S_{\pm} = S_x \pm iS_y$ when an external magnetic field is introduced. The dynamics may be handled perturbatively in the weak-coupling domain, and the field's primary structure of quantum coherence is captured by the first-order approximation.

At this level, the coupling between the spin and the magnetic field is encoded in the interaction strength parameter, and the time-evolved intelligent coherent states are achieved by keeping just linear contributions. The states that result are provided by

$${}^{1a} |n_1(t)\rangle = e^{-i\frac{E_1 t}{\hbar}} {}^{(1)} \left\{ \omega_1 |0\rangle + |1\rangle \right\} \quad (3.6)$$

$${}^{1a} |n_2(t)\rangle = e^{-i\frac{E_0 t}{\hbar}} {}^{(1)} \left\{ |-1\rangle + |0\rangle + \omega_1 |1\rangle \right\} \quad (3.7)$$

$${}^{1a} |n_3(t)\rangle = e^{-i\frac{E_{-1} t}{\hbar}} {}^{(1)} \left\{ |-1\rangle + \omega_1 \left(\frac{e^{i\omega t}}{\omega + \omega_0} - \frac{e^{-i\omega t}}{\omega - \omega_0} \right) |0\rangle \right\} \quad (3.8)$$

These formulations unequivocally demonstrate that coherent superpositions of the basic states utilized in the $\text{su}_f(2)$ formalism are induced by the external magnetic field. In the spin Hilbert space, the magnetic interaction causes the previously localized spin states to change into delocalized quantum states. This

evolution and the ensuing quantum coherence can be conveniently described algebraically using the $\mathfrak{su}_f(2)$ framework.

Physically speaking, the magnetic field causes coherent transitions between the three spin-1 magnetic sublevels; the strength of the field and frequency detuning define the amplitude of these transitions. The system mostly stays in its initial state in the weak-field regime, but quantum transitions cause tiny populations to move to nearby spin states. Quantum interference effects between the magnetic sublevels are produced by the $\mathfrak{su}_f(2)$ symmetry, which also maintains the angular momentum structure and produces coherent rotations of the spin state. Within the spin subspace, the ensuing coherent superposition produces entanglement-like characteristics and quantum correlations. As a result, the first-order $\mathfrak{su}_f(2)$ approximation offers the fundamental mechanism for the creation of intelligent coherent spin states and provides a basis for studying nonlinear phenomena including improved quantum correlations, population redistribution, and spin squeezing.

3.3. Second-Order Approximation within the $\mathfrak{su}_f(2)$ Formalism

The second-order contributions resulting from the interaction between the spin-1 particle and the external magnetic field must be taken into account in order to provide a more precise description of the spin dynamics.

These corrections come from repeated actions of the ladder operators S_+ and S_- within the $\mathfrak{su}_f(2)$ formalism, which produce more couplings between the magnetic sublevels. As a result, the interaction parameter of the emerging intelligent coherent states has quadratic terms, indicating the beginning of nonlinear quantum effects.

One way to express the second-order coherent states is as

$${}^{2a} |n_1(t)\rangle = e^{-i\frac{E_1 t}{\hbar}} {}^{(2)} \left\{ \omega_1 |-1\rangle + (1 - \omega_1^2) |0\rangle + a |1\rangle \right\} \quad (3.9)$$

$${}^{2a} |n_2(t)\rangle = e^{-i\frac{E_0 t}{\hbar}} {}^{(2)} \left\{ |-1\rangle + |0\rangle + (1 + \omega_1^2) |1\rangle \right\} \quad (3.10)$$

$${}^{2a} |n_3(t)\rangle = e^{-i\frac{E_0 t}{\hbar}} {}^{(2)} \left\{ (1 + \omega_1^2 n_1) |-1\rangle - n_2 |0\rangle + n_3 |1\rangle \right\} \quad (3.11)$$

The spin-state amplitudes in the second-order approximation have quadratic corrections proportional to ω_1^2 , indicating that the external magnetic field has a greater impact on the dynamics of the system. Deviations from the linear behavior expected by the first-order approximation occur as ω_1^2 rises due to the considerable redistribution of the populations of the magnetic sublevels $|1\rangle$, $|0\rangle$, and $|-1\rangle$.

3.4. Third-Order Approximation within the $\mathfrak{su}_f(2)$ Formalism

The perturbative expansion of the Hamiltonian must take into account the third-order contributions in order to thoroughly examine the nonlinear behavior of the spin-1 system interacting with an external magnetic field. The generators S_x , S_y , and S_z control the quantum dynamics inside the $\mathfrak{su}_f(2)$ framework by operating on the base states $|1\rangle$, $|0\rangle$, and $|-1\rangle$, which correspond to the eigenvalues $m = +1, 0, -1$ of the spin projection operator S_z .

The linear and quadratic couplings between the magnetic sublevels are described by the first- and second-order approximations. Nevertheless, higher-order transition processes produced by repeated applications of the $\mathfrak{su}_f(2)$ formalism to the ladder operators S_+ and S_- are introduced by the third-order approximation. The coherent spin states evolve in a substantially nonlinear manner as a result of these processes, which produce cubic powers of the interaction parameter.

The corresponding third-order intelligent coherent states are obtained as

$${}^3 |n_1(t)\rangle = e^{-i\frac{\theta}{\hbar} t} {}^{(3)} \left\{ |-1\rangle + [1 + \omega_1^2 + \omega_1^3] |0\rangle + [\omega_1 + \omega_1^3] |1\rangle \right\} \quad (3.12)$$

$${}^3 |n_2(t)\rangle = e^{-i\frac{\theta}{\hbar} t} {}^{(3)} \left\{ \omega_1^4 |-1\rangle + \omega_1^3 |0\rangle \right\} \quad (3.13)$$

$${}^3 |n_3(t)\rangle = e^{i\frac{\theta}{\hbar} t} {}^{(3)} \left\{ \omega_1^4 |-1\rangle + \frac{\beta}{\theta} \omega_1^3 |0\rangle - \omega_1^4 |1\rangle \right\} \quad (3.14)$$

A severely nonlinear phase in the spin-1 dynamics is revealed by the third-order approximation when the interaction parameter ω_1^2 grows. Multiple virtual transitions between magnetic sublevels and subsequent spin-field interactions are the source of the higher-order components, which are proportional to ω_1^3 and ω_1^4 . As a result, compared to the first-order approximation, there is a more complicated and asymmetric redistribution of the populations of $|1\rangle$, $|0\rangle$, and $|-1\rangle$. A greater area of $\mathfrak{su}_f(2)$ space can be explored by the state due to the enhancement of quantum interference effects caused by an increase in ω_1^2 . This leads to a considerable dependence of the expected values and variances of the spin operators on the strength of the interaction. Significant changes to the uncertainty relations result in reduced quantum noise in some spin components and increased spin squeezing. Additionally, the coherence between various spin projections is strengthened, leading to greater entanglement and stronger quantum correlations. The transition probabilities stop growing linearly with the strength of the magnetic field when ω_1^2 is strong enough to cause saturation effects. As a result, ω_1^2 plays a crucial role in regulating the establishment of extremely nonclassical intelligent coherent states and the departure from conventional $\mathfrak{su}_f(2)$ coherent states.

4. Results and Discussion

The current findings demonstrate that nonlinear coherent dynamics are produced inside the $\mathfrak{su}_f(2)$ framework when a spin-1 particle interacts with an external magnetic field. Coherent superposition states, whose complexity rises with higher-order corrections, are created when the magnetic field connects the three spin projections, $|1\rangle$, $|0\rangle$, and $|-1\rangle$. Simple transitions between adjacent levels are described by the first-order approximation, while additional transition paths and multiphoton-like processes are introduced by the second- and third-order terms.

Quantum interference effects and population transfer are enhanced by the corresponding contributions, which are proportional to ω_1^2 , ω_1^3 and ω_1^4 . In the Hilbert space, the spin state gets progressively delocalized as ω_1^2 grows, strengthening off-diagonal density-matrix components and quantum coherence. Additionally, the nonlinear interaction changes the uncertainty relations of the spin operators and produces higher quantum correlations. Larger entanglement and improved spin squeezing follow, exhibiting clear nonclassical behavior. Saturation effects become prominent and the dynamics diverge from typical $\mathfrak{su}_f(2)$ coherent states for bigger contact strengths and longer evolution durations. These findings show that highly correlated intelligent coherent states with rich quantum features can be efficiently created using nonlinear spin-field coupling.

Quantum correlations, entanglement measurements, and uncertainty relations were used to examine the nonclassical features of the intelligent coherent spin states. The findings demonstrate that these states have greater quantum correlations than regular $SU(2)$ coherent states and satisfy the generalized minimum-uncertainty criterion. Spin squeezing and the redistribution of quantum fluctuations result from nonlinear spin-field interactions that alter the variances of the spin components.

Coherence between the magnetic sublevels is improved as the interaction strength rises, leading to greater entanglement and more noticeable nonclassical behavior. The significance of higher-order effects is highlighted by the third-order approximation, which shows the biggest departures from classical dynamics. Because of these characteristics, intelligent coherent spin states are attractive options for high-precision quantum metrology, quantum information, and quantum computation.

5. Conclusion

The creation and characteristics of intelligent coherent states for a spin-1 system interacting with an external magnetic field under the $\mathfrak{su}_f(2)$ formalism have been examined in this work. We have examined the impact of nonlinear spin-field couplings on the coherent states' structure and quantum statistical characteristics by evolving the Hamiltonian up to third order in the interaction parameter. The acquired results demonstrate that coherent transitions between the three magnetic sublevels $|1\rangle$, $|0\rangle$, and $|-1\rangle$ are induced by the external magnetic field, resulting in the production of generalized intelligent coherent states. While the second- and third-order corrections show the appearance of nonlinear effects that drastically alter the

population distribution and the evolution of the quantum state, the first-order approximation explains the system's basic coherent dynamics.

These states meet the generalized minimum-uncertainty requirement, according to a thorough examination of the expectation values, variances, and uncertainty relations. As the interaction intensity and perturbation order rise, the nonlinear interaction produces nonclassical phenomena like spin squeezing and improved quantum coherence, which become more noticeable.

The emergence of substantial quantum correlations brought about by the magnetic-field interaction is one of the study's most significant findings. Highly coherent superpositions are produced by the interaction between the spin projections and cannot be explained by a conventional framework. The emergence of quantum entanglement, a key resource for quantum technology, coincides with these correlations. The degree of entanglement rises as the Hamiltonian's nonlinear contributions do, suggesting that higher-order spin-field interactions are essential for the creation of nonclassical quantum states.

The relationship between spin squeezing, quantum coherence, and entanglement highlights the rich physical structure of the intelligent coherent states. The third-order approximation, in particular, exhibits the strongest quantum correlations and the most significant deviations from ordinary $su_f(2)$ coherent states, emphasizing the importance of nonlinear effects in the dynamics of spin systems.

These results show that intelligent coherent states offer an effective paradigm for examining how non-linearity, quantum entanglement, and quantum correlations interact in finite-dimensional systems. Beyond their basic interest, these states could have significant uses in nonlinear quantum optics, quantum information processing, quantum computation, quantum communication, and quantum metrology, all of which depend on the creation and management of entangled states.

The quantitative assessment of entanglement measures, such as the von Neumann entropy, linear entropy, or concurrence-like quantities modified for spin systems, as well as the impact of decoherence and environmental interactions on the robustness of the produced quantum correlations, may be the subject of future research. Deeper understanding of the function of intelligent coherent states as useful resources for the advancement of next-generation quantum technology may result from such investigations.

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